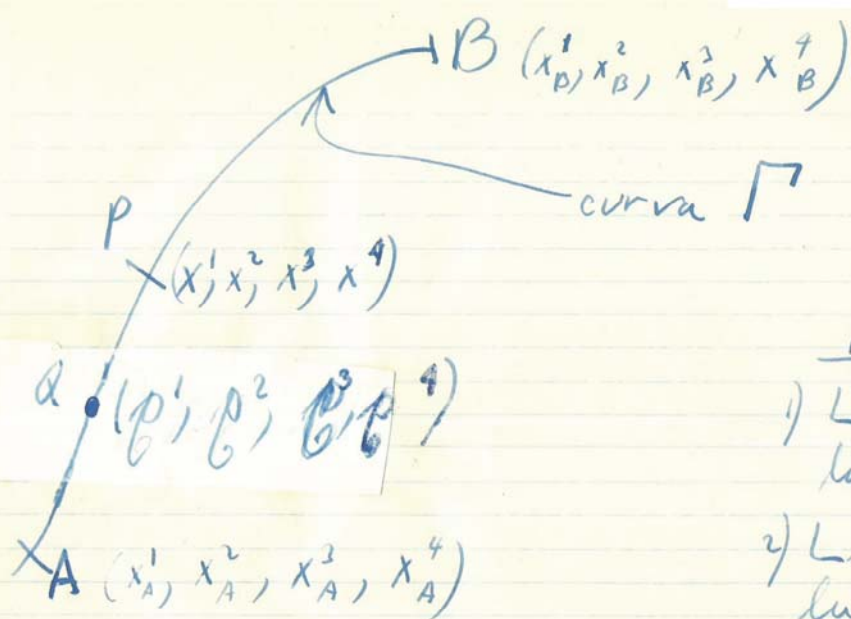

Cuaderno de notas de trabajo

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Cuaderno Sn 5



Notación:

- 1) Letras góticas para las funciones de Q.
- 2) Letras romanas para las funciones de P.

$$G_{ij}(x^1, x^2, x^3, x^4); \quad g_{ij}(p^1, p^2, p^3, p^4)$$

G_{ij}, g_{ij} tensor de dos índices (covariantes)

$$J = \int_A^B \left[\int_A^P g_{ij} dp^j \right] dx^i$$

$$\delta J = \int_A^B \left\langle \delta \left[\int_A^P g_{ij} dp^j \right] \right\rangle dx^i + \left[\int_A^P g_{ij} dp^j \right] \delta(dx^i)$$

$$(\delta J)_1 = \int_A^B \left\langle \delta \left[\int_A^P g_{ij} dp^j \right] \right\rangle dx^i \quad (\delta J)_2 = \left[\int_A^P g_{ij} dp^j \right] \delta(dx^i)$$

$$\delta J = \int_A^B \left[\frac{\partial g_{ij}}{\partial p^k} \delta p^k dp^j + g_{ij} d(\delta p^j) \right] dx^i$$

$$\delta J = \int_A^B$$

$$\delta \int_A^P g_{ij} dp^j = \int_A^P \left[\frac{\partial g_{ij}}{\partial p^k} \delta p^k dp^j + g_{ij} d(\delta p^j) \right]$$

$$\delta \int_A^P g_{ij} dp^j = g_{ij} \delta p^j \Big|_A^P + \int_A^P \left[\frac{\partial g_{ij}}{\partial p^k} \delta p^k dp^j - \frac{\partial g_{ij}}{\partial p^p} \delta p^i dp^p \right]$$

$$\delta \int_A^P g_{ij} dp^j = G_{ij} \delta x^j + \int_A^P \left[\frac{\partial g_{ia}}{\partial p^b} - \frac{\partial g_{ib}}{\partial p^a} \right] \delta p^b dp^a$$

$$\delta J_1 = \int_A^B G_{ij} \delta x^j dx^i + \int_A^P \left[\frac{\partial g_{ia}}{\partial p^b} - \frac{\partial g_{ib}}{\partial p^a} \right] \delta p^b dp^a$$

$$\delta J_1 = \int_A^B G_{ij} \delta x^j dx^i + \int_A^B \left[\frac{\partial g_{ia}}{\partial p^b} - \frac{\partial g_{ib}}{\partial p^a} \right] \delta p^b dp^a dx^i$$

$$(\delta J)_2 = \int_A^B \left\langle \int_A^P g_{ij} dp_j^i \right\rangle d(x^i)$$

$$(\delta J)_2 = \left\langle \int_A^P g_{ij} dp_j^i \right\rangle \delta x^i \Big|_A^B - \int_A^B G_{ij} dx^j \delta x^i$$

$$\delta J_2 = - \int_A^B G_{ij} dx^j \delta x^i$$

Hipótesis: G_{ij}, g_{ij} tensor simétrico.

$$\delta J = \int_A^B \left\langle \int_A^P \left[\frac{\partial g_{ij}}{\partial p^k} - \frac{\partial g_{ik}}{\partial p^j} \right] \frac{dp^k}{dp^i} \right\rangle dx^i$$

$$J = \int_A^B \left\langle \int_A^P g_{ij} dp_j^i \right\rangle dx^i$$

$$\Delta_{ij}, h_{ij}, v^i, dx^i, \boxed{\frac{\partial h_{ij}}{\partial x^k}}$$

$$a_i = \frac{\partial h_{ij}}{\partial x^k} v^j v^k - \frac{\partial h_{jk}}{\partial x^i} v^j v^k$$

$$h^{ip} a_i = h^{ip} \frac{\partial h_{ij}}{\partial x^k} v^j v^k - h^{ip} \frac{\partial h_{jk}}{\partial x^i} v^j v^k$$

$$\frac{\partial h_{ij}}{\partial x^k} v^j - \frac{\partial h_{jk}}{\partial x^i} v^j = T_{ik}$$

$$\cancel{h^{ik}} h^{ik} \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right) v^j = h^{ik} T_{ik}$$

$$S = \int_A^B \sqrt{\Delta_{ij} dx^i dx^j} ; \quad \Sigma = \int_A^B \sqrt{h_{ij} dx^i dx^j}$$

$$J = S - \Sigma \quad \delta J = 0$$

$$\delta J = \int_A^B \frac{\Delta_{ij} dx^i d(\delta x^j)}{ds} - \int_A^B \frac{h_{ij} dx^i d(\delta x^j)}{\sqrt{h_{ij} dx^i dx^j}}$$

$$\delta J = - \int_A^B a_j \delta x^j ds + \int_A^B \frac{d}{ds} \left(\frac{h_{ij} v^i}{\sqrt{h_{ij} v^i v^j}} \right) (\delta x^j)$$

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$$\Delta^{kl} \frac{\partial^2 h_{ij}}{\partial x^k \partial x^l} v^i v^j$$

$$\frac{\partial h_{ij}}{\partial x^k} v^i v^j$$

$$\delta J = - \int_A^B a_j \delta x^j ds + \int_A^B \frac{\sqrt{h_{ij} v^i v^j} \delta h_{ij}}{\sqrt{h_{ij} v^i v^j}}$$

$$S_{(n)} = h_{p_1}^{i_1} h_{p_2}^{i_2} h_{p_3}^{i_3} \dots h_{p_{n-1}}^{i_{n-1}} h_{p_n}^{i_n} v_i^1 v_i^2 \dots v_i^n$$

$$v^i = \frac{dx^i}{ds}$$

$$ds = \sqrt{\Delta_{ab} dx^a dx^b}$$

$$ds = \sqrt{\Delta_{ab} dx^a dx^b}$$

$$\delta(ds) = \frac{\Delta_{ab} dx^a d(\delta x^b)}{ds^2} = \Delta_{ab} \frac{dx^a}{ds} d(\delta x^b)$$

$$\delta(ds) = v_a d(\delta x^a)$$

$$\delta v^i = \frac{d}{ds}(\delta x^i) - \frac{dx^i}{ds^2} v_a d(\delta x^a)$$

$$\delta v^i = \frac{d}{ds}(\delta x^i) - v^i v_a d(\delta x^a)$$

$$\delta(v_j) = \frac{d}{ds}(\delta x_j) - v_j v_a d(\delta x^a)$$

$$\delta(h_{p_{k+1}}^{p_k}) = \frac{\partial h_{p_{k+1}}^{p_k}}{\partial x^c} \delta x^c$$

$$\begin{aligned} \delta S_{(n)} = & \left[\frac{\partial h_{p_1}^i}{\partial x^c} h_{p_1}^{p_1} h_{p_2}^{p_2} - h_{p_n}^{p_{n-1}} h_j^{p_n} v_i v_j \delta x^c \right. \\ & + h_{p_1}^i h_{p_2}^{p_1} h_{p_3}^{p_2} - h_{p_n}^{p_{n-1}} \frac{\partial h_j^{p_n}}{\partial x^c} v_i v_j \delta x^c \\ & + \sum_{k=1}^{n-1} h_{p_1}^i h_{p_2}^{p_1} - h_{p_{k+1}}^{p_k} \frac{\partial h_{p_{k+1}}^{p_k}}{\partial x^c} - h_j^{p_n} v_i v_j \frac{\partial h_{p_{k+1}}^{p_k}}{\partial x^c} \delta x^c \\ & + \end{aligned}$$

$$\delta g_{ij} = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right) v^j$$

$$\delta \int_A^B h_{ij} dx^i = \int \left[\frac{\partial h_{ij}}{\partial x^k} \delta x^k dx^i + h_{ij} d(\delta x^i) \right]$$

$$\delta \int_A^B h_{ij} dx^i = \int_A^B \left[\frac{\partial h_{ij}}{\partial x^k} dx^j \delta x^k - \frac{\partial h_{ij}}{\partial x^k} dx^k \delta x^j \right]$$

$$\boxed{\delta \int_A^B h_{ij} dx^i = \int_A^B \left[\frac{\partial h_{ij}}{\partial x^k} dx^j - \frac{\partial h_{ik}}{\partial x^j} dx^j \right] \delta x^k}$$

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$$\left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^j} \right) v^j = T_{ik}$$

$$\frac{dT_{ik}}{ds} = \left(\frac{\partial^2 h_{ij}}{\partial x^k \partial x^p} - \frac{\partial^2 h_{ik}}{\partial x^j \partial x^p} \right) v^j v^p + \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^j} \right) a^j$$

$$\frac{dT_{ik}}{ds} = \left(\frac{\partial^2 h_{ij}}{\partial x^k \partial x^p} - \frac{\partial^2 h_{ik}}{\partial x^j \partial x^p} \right) v^j v^p + \Delta^{jr} \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^j} \right) \left(\frac{\partial h_{rs}}{\partial x^t} - \frac{\partial h_{st}}{\partial x^r} \right) v^s v^t$$

$$\frac{dT_{ik}}{ds} = \left[\left(\frac{\partial^2 h_{ij}}{\partial x^k \partial x^p} - \frac{\partial^2 h_{ik}}{\partial x^j \partial x^p} \right) \Delta^{jr} \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{ik}}{\partial x^j} \right) \left(\frac{\partial h_{rs}}{\partial x^t} - \frac{\partial h_{st}}{\partial x^r} \right) \right] v^s v^t$$

$$\Delta^{ij} h_{ij}, h_{ij} v^i v^j,$$

$$(1-\gamma z)^{-\frac{1}{2}} = 1 + \frac{1}{2}\gamma z + \frac{1}{2} \cdot \frac{3}{4}\gamma^2 z^2 + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6}\gamma^3 z^3 + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8}\gamma^4 z^4 + \dots$$

$$(1-\gamma^{-1}z)^{-\frac{1}{2}} = 1 + \frac{1}{2}\gamma^{-1}z + \frac{1}{2} \cdot \frac{3}{4}\gamma^{-2}z^2 + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6}\gamma^{-3}z^3 + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8}\gamma^{-4}z^4 + \dots$$

$$(1-\gamma z)(1-\gamma^{-1}z)^{-\frac{1}{2}} = \left[\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \dots - \frac{(2n-3)}{(2n-2)} \frac{(2n-1)}{2n} (\gamma^n + \gamma^{-n}) + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{4} \right. \\ \left. + \frac{1}{2} \left(\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \right) \cdot \left(\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \dots - \frac{(2n-7)}{(2n-6)} \right) (\gamma^{n-6} + \gamma^{-n}) \right]$$

$$\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \dots (2n-2)(2n-1)(2n)}{2^n \cdot n! \cdot 2^n \cdot n!} \gamma^{-n} \left[1 + \frac{1}{2} \cdot \frac{2n}{2n-1} \gamma^2 + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{2n}{(2n-1)} \right. \\ \left. 1 \cdot 2^{2n} + \frac{1}{2} \cdot \frac{2n}{(2n-1)} \gamma^{2n-2} + \frac{1}{2} \cdot \frac{3}{4} \right]$$

$$+ \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \dots \frac{(2n-3)(2n-1)}{(2n-2)2n} z^n + \dots$$

$$+ \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \dots \frac{(2n-1)(2n-1)}{(2n-2)2n} z^{-n} + \dots$$

$$\frac{5}{6} \dots \frac{(2n-3)}{(2n-4)} (z^{n-2} + z^{-n+2}) + \frac{1}{2} \cdot \frac{3}{4} \cdot \left(\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \dots \frac{(2n-5)}{(2n-4)} (z^{n-4} + z^{-n+4}) \right)$$

$$+ \dots$$

$$\frac{(2n-2)}{(2n-3)} z^4 + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \frac{2n(2n-2)(2n-4)}{(2n-1)(2n-3)(2n-5)} z^6 + \dots$$

$$\frac{2n}{(2n-1)} \cdot \frac{(2n-2)}{(2n-3)} z^{2n-4} + \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{2n(2n-2)(2n-4)}{(2n-1)(2n-3)(2n-5)} z^{2n-6} + \dots$$

$$F\left(\frac{1}{2}, -n, \frac{1}{2} - n, y^2\right)$$

$$[F] = \frac{1}{2}(-n)$$

$$F = 1 + \frac{\frac{1}{2} \cdot (-n)}{1 \cdot (\frac{1}{2} - n)} y^2 + \frac{\frac{1}{2} \cdot (\frac{3}{2}) \cdot (-n) \cdot (-n+1)}{1 \cdot 2 \cdot (\frac{1}{2} - n) \cdot (\frac{3}{2} - n)} y^4 + \frac{\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2} \cdot (-n) \cdot (-n+1) \cdot (-n+2)}{1 \cdot 2 \cdot 3 \cdot (\frac{1}{2} - n) \cdot (\frac{3}{2} - n) \cdot (\frac{5}{2} - n)} y^6$$

$$F = 1 + \frac{1}{2} \cdot \frac{2n}{2n-1} y^2 + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{2n(2n-2)}{(2n-1)(2n-3)} y^4 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{2n(2n-2)(2n-4)}{(2n-1)(2n-3)(2n-5)} y^6$$

$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdot 8 \cdots 2n} \cdot \frac{2n(2n-2)(2n-4) \cdots 2}{(2n-1)(2n-3) \cdots 1} y^{2n}$$

$$2n-8+2$$

$$2n-8+1$$

$$2n-(2n-2)+2$$

$$y^6 + \frac{\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{5}{2} \cdot \frac{7}{2} \cdot (-n)(-n+1)(-n+2)(-n+3)}{1 \cdot 2 \cdot 3 \cdot 4 \left(\frac{1}{2}-n\right)\left(\frac{3}{2}-n\right)\left(\frac{5}{2}-n\right)\left(\frac{7}{2}-n\right)} y^8 + \dots$$

$$y^6 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \frac{2n(2n-2)(2n-4)(2n-6)}{(2n-1)(2n-3)(2n-5)(2n-7)} y^8 + \dots$$

$$+ \frac{1 \cdot 3 \cdot 5 \dots (2n-3) \cdot 2n(2n-2)(2n-4) \dots 4}{2 \cdot 4 \cdot 6 \dots (2n-2) \cdot (2n-1)(2n-3) \dots 3} y^{2n-2}$$

$$P_n(\mu) = \frac{(2n)!}{2^{2n} (n!)^2} \left[F\left(\frac{1}{2}, -n, \frac{1}{2}-n, 2\mu^2\right) \right]$$

$$\mu P'_n(\mu) - P'_{n-1}(\mu) = n P_n(\mu) \quad (35)$$

$$P'_{n+1}(\mu) - \mu P'_n(\mu) = (n+1) P_n(\mu) \quad (36)$$

$$n P'_{n+1}(\mu) - (2n+1) \mu P'_n(\mu) + (n+1) P'_{n-1}(\mu) = 0 \quad \times$$

$$\boxed{\mu P'_n(\mu) = \varphi_n}$$

$$\varphi_n - P'_{n-1} - n P_n = 0$$

$$-\varphi_n - P'_{n+1} - (n+1) P_n = 0$$

$$n P'_{n+1} - (2n+1) \varphi_n + (n+1) P'_{n-1} - (n+1)$$

$$q_n - p'_{n-1} \cdot \quad -n p_n = 0$$

$$-q_n \quad + p'_{n+1} - (n+1)p_n = 0$$

$$-(n+1)q_n + (n+1)p'_{n-1} + n p'_{n+1} = 0$$

$$\begin{vmatrix} 1 & -1 & -n p_n \\ -1 & 0 & (+p'_{n+1} - (n+1)p_n) \\ -(n+1) & (n+1) & +n p'_{n+1} \end{vmatrix} = 0$$

$$\underline{n(n+1)p_n} + (-p'_{n+1} + (n+1)p_n) \underset{-n}{((n+1) - (2n+1))} + n p'_{n+1} (-1) = 0$$

$$n(n+1)P_n - n(-P'_{n+1} + (n+1)P_n) - n(P'_{n+1}) = 0$$

$$\cancel{-2nP'_{n+1}} = \cancel{[-n(n+1) + n(n+1)P_n]}$$

$$(37) \quad (1-\mu^2)P'_n(\mu) = nP_{n-1}(\mu) - n\mu P_n(\mu) \quad \left| \begin{matrix} (n+1) \\ + \end{matrix} \right.$$

$$(37') \quad (1-\mu^2)P'_n(\mu) = (n+1)\mu P_n(\mu) - (n+1)P_{n+1}(\mu) \quad \left| \begin{matrix} n \\ - \end{matrix} \right.$$

$$(2n+1)(1-\mu^2)P'_n(\mu) = n(n+1)P_{n-1}(\mu) - n(n+1)P_{n+1}(\mu)$$

$$(2n+1)(1-\mu^2)P'_n(\mu) = n(n+1)[P_{n-1}(\mu) - P_{n+1}(\mu)]$$

$$\int_{-1}^{+1} (\mu^2 - 1) P_{n+1} P_n' d\mu = \int_{-1}^{+1} P_{n+1} P_{n-1} d\mu - n(n+1) \int_{-1}^{+1} P_{n+1}^2 d\mu$$

$$\int_{-1}^{+1} (1 - \mu^2) P_{n+1} P_n' d\mu = - \frac{n(n+1)}{(2n+1)} \int_{-1}^{+1} P_{n+1}^2 d\mu$$

$$\int_{-1}^{+1} (\mu^2 - 1) P_{n+1}(\mu) P_n'(\mu) d\mu = \frac{n(n+1)}{(2n+1)} \frac{1}{(2n+3)} = \frac{2n(n+1)}{(2n+1)(2n+3)}$$

n even

$$\int_0^1 \mu^m P_n d\mu = \frac{m(m-2)(m-4) \dots (m-n+2)}{(m+n+1)(m+n-1)(m+n-3) \dots (m+1)}$$

$$\int_0^1 \mu^{m-1} P_{n-1} d\mu = \frac{(m-2)(m-4) \dots (m-n+2)}{(m+n-1)(m+n-3) \dots (m+1)}$$

$$(m+n+1) \int_0^1 \mu^m P_n(\mu) d\mu = m \int_0^1 \mu^{m-1} P_{n-1}(\mu) d\mu$$

$$\frac{m}{m+n+1}$$

n odd

$$\int_0^1 \mu^m P_n d\mu = \frac{m(m-3)\dots(m-n+2)}{(m+n+1)(m+n-1)\dots(m+2)}$$

$$\int_0^1 \mu^{m-1} P_{n-1} d\mu = \frac{(m-1)(m-3)\dots(m-n+2)}{(m+n-1)(m+n-3)\dots(m+1)}$$

$$(m+n+1) \int_0^1 \mu^m P_n(\mu) d\mu = m \int_0^1 \mu^{m-1} P_{n-1}(\mu) d\mu$$

n even ($n-2$, even)

$$\int_0^1 \mu^m P_n(\mu) d\mu = \frac{m(m-2)\dots(m-n+2)}{(m+n+1)(m+n-1)\dots(m+1)}$$

$$\int_0^1 \mu^m P_{n-2}(\mu) d\mu = \frac{m(m-2)\dots(m-n+4)}{(m+n-1)(m+n-3)\dots(m+1)}$$

$$(m+n-1) \int_0^1 \mu^m P_n(\mu) d\mu = (m-n+2) \int_0^1 \mu^m P_{n-2}(\mu) d\mu$$

n odd $(n-2, \text{odd})$

$$\int_0^1 \mu^m P_n(\mu) d\mu = \frac{(m-1)(m-3) \cdots (m-n+2)}{(m+n+1)(m+n-1) \cdots (m+2)}$$

$$\int_0^1 \mu^m P_{n-2}(\mu) d\mu = \frac{(m-1)(m-3) \cdots (m-n+4)}{(m+n-1)(m+n-3) \cdots (m+2)}$$

$$(m+n+1) \int_0^1 \mu^m P_n(\mu) d\mu = (m-n+2) \int_0^1 \mu^m P_{n-2}(\mu) d\mu$$

$$\frac{dx^i}{ds} = \dot{x}^i$$

$$a_i = \left(\frac{\partial h_{ij}}{\partial x^k} - \frac{\partial h_{jk}}{\partial x^i} \right) \frac{dx^j}{ds} \frac{dx^k}{ds}$$

$$a_i = \left(\frac{\partial h_{ij}}{\partial x^k} \frac{dx^k}{ds} \right) \frac{dx^j}{ds} - \frac{\partial h_{jk}}{\partial x^i} \frac{dx^j}{ds} \frac{dx^k}{ds}$$

$$a_i = \left(\frac{d}{ds} (h_{ij}) \right) \frac{dx^j}{ds} - \frac{\partial h_{jk}}{\partial x^i} \frac{dx^j}{ds} \frac{dx^k}{ds}$$

$$a_i = \frac{d}{ds} \left\{ h_{ij} \frac{dx^j}{ds} \right\} - \frac{\partial}{\partial x^i} \left\{ h_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} \right\} - h_{ij} \ddot{x}^j$$

$$\delta \int_A^B h_{ij} dx^i = \int_A^B \left[\frac{\partial h_{ij}}{\partial x^k} \delta x^k dx^i + h_{ij} d(\delta x^i) \right]$$

$$\delta \int_A^B h_{ij} dx^i =$$

$$I = \int_A^B h_{jk} \frac{dx^i}{d\tau} dx^k$$

$$\delta I = \int_A^B \left[\frac{\partial h_{jk}}{\partial x^k} \delta x^k \frac{dx^i}{d\tau} dx^k + h_{jk} \right]$$

$$a_i + h_{ij} a^j = \frac{d}{ds} \{ h_{ij} v^j \} - \frac{\partial}{\partial x^i} \{ h_{jk} v^j v^k \}$$

$$(\Delta_{ij} + h_{ij}) a^j = \frac{d}{ds} \{ h_{ij} v^j \} - \frac{\partial}{\partial x^i} \{ h_{jk} v^j v^k \}$$

$$h_{ij} v^i a^j = v^i \frac{d}{ds} \{ h_{ij} v^j \} - \frac{d}{ds} \{ h_{jk} v^j v^k \}$$

$$= \cancel{\frac{d}{ds} \{ h_{ij} v^i v^j \}} - h_{ij} a^i v^j - \cancel{\frac{d}{ds} \{ h_{jk} v^j v^k \}}$$

$$I = \int_0^{\pi} \frac{d\varphi}{1 + \xi \cos \varphi}$$

$$I = \int_0^{\pi} \frac{d\varphi}{(1+\xi)\cos^2\frac{\varphi}{2} + (1-\xi)\sin^2\frac{\varphi}{2}}$$

$$I = \int_0^{\pi} \frac{\sec^2\frac{\varphi}{2} d\varphi}{(1+\xi) + (1-\xi)\tan^2\frac{\varphi}{2}}$$

$$t = \tan\frac{\varphi}{2}$$

$$I = 2 \int_0^{\infty} \frac{dt}{(1+\xi) + (1-\xi)t^2}$$

$$I = \frac{2}{1+\xi} \int_0^{\infty} \frac{dt}{1 + \frac{1-\xi}{1+\xi} t^2}$$

$$I = \frac{2}{(1+\xi)\sqrt{1-\xi}} \int_0^{\infty} \frac{\sqrt{\frac{1-\xi}{1+\xi}} dt}{1 + \left(\sqrt{\frac{1-\xi}{1+\xi}} t\right)^2}$$

$$I = \frac{2}{\sqrt{1-\xi^2}} \tan^{-1} \left(\sqrt{\frac{1-\xi}{1+\xi}} t \right)$$

$$I = \frac{\pi}{\sqrt{1-\xi^2}}$$

$$\xi = \mp \alpha \sqrt{\frac{z^2-1}{1-\alpha^2 z^2}}$$

$$\xi = \mp \alpha \frac{\sqrt{z^2-1}}{1-\alpha^2 z^2}$$

$$1-\xi^2 = 1 - \alpha^2 \frac{z^2-1}{(1-\alpha^2 z^2)^2}$$

$$1-\xi^2 = \frac{1-2\alpha^2 z^2 + \alpha^4 z^4 - \alpha^2 z^2 + \alpha^2}{(1-\alpha^2 z^2)^2}$$

$$1-\xi^2 = \frac{(1-\alpha^2 z^2)^2}{(1-\alpha^2 z^2)^2} = 1$$

$$\xi = \mp \alpha \frac{\sqrt{z^2 - 1}}{1 - \alpha z}$$

$$1 - \xi^2 = 1 - \alpha^2 \frac{z^2 - 1}{(1 - \alpha z)^2}$$

$$1 - \xi^2 = \frac{1 - 2\alpha z + \alpha^2 z^2 - \alpha^2 z^2 + \alpha^2}{(1 - \alpha z)^2}$$

$$\sqrt{1 - \xi^2} = \frac{\sqrt{1 - 2\alpha z + \alpha^2}}{1 - \alpha z}$$

$$1 + \xi \cos \psi = 1 \mp \alpha \frac{\sqrt{z^2 - 1}}{1 - \alpha z} \cos \psi$$

$$1 + \xi \cos \psi = \frac{1 - \alpha z \mp \alpha \sqrt{z^2 - 1} \cos \psi}{1 - \alpha z}$$

$$\mathcal{L} = A_{\alpha_1 \alpha_2} v^{\alpha_1} v^{\alpha_2} + A_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} v^{\alpha_1} v^{\alpha_2} v^{\alpha_3} v^{\alpha_4} + \dots$$

$$\frac{\partial \mathcal{L}}{\partial v^k} = \left[A_{k \alpha_2} v^{\alpha_2} + A_{\alpha_1 k} v^{\alpha_1} \right. \\ \left. + A_{k \alpha_2 \alpha_3 \alpha_4} v^{\alpha_2} v^{\alpha_3} v^{\alpha_4} + A_{\alpha_1 k \alpha_2 \alpha_3 \alpha_4} v^{\alpha_1} v^{\alpha_2} v^{\alpha_3} v^{\alpha_4} \right. \\ \left. + A_{\alpha_1 \alpha_2 k \alpha_4} v^{\alpha_1} v^{\alpha_2} v^{\alpha_4} + A_{\alpha_1 \alpha_2 \alpha_3 k} v^{\alpha_1} v^{\alpha_2} v^{\alpha_3} \right. \\ \left. + \dots \right]$$

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$$\frac{d}{ds} \frac{\partial \mathcal{L}}{\partial v^k} = \frac{\partial A_{k\alpha_2}}{\partial x^p} v^p v^{\alpha_2} + A_{k\alpha_2} a^{\alpha_2} + \frac{\partial A_{\alpha_1 k}}{\partial x^p} v^p v^{\alpha_1} + A_{\alpha_1 k} a^{\alpha_1}$$

$$+ \frac{\partial A_{k\alpha_2\alpha_3\alpha_4}}{\partial x^p} v^p v^{\alpha_2} v^{\alpha_3} v^{\alpha_4} + A_{k\alpha_2\alpha_3\alpha_4} a^{\alpha_2} v^{\alpha_3} v^{\alpha_4}$$

$$+ A_{k\alpha_2\alpha_3\alpha_4} v^{\alpha_2} a^{\alpha_3} v^{\alpha_4} + A_{k\alpha_2\alpha_3\alpha_4} v^{\alpha_2} v^{\alpha_3} a^{\alpha_4}$$

$$+ \partial A_{\alpha_1 \alpha}$$

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$$\int_A^B \left\{ \int_A^B F dx \left\{ \frac{\partial G}{\partial y} - \frac{d}{ds} \left(\frac{\partial G}{\partial y'} \right) \right\} + \int_A^B G dx \left\{ \frac{\partial F}{\partial y} - \frac{d}{ds} \left(\frac{\partial F}{\partial y'} \right) \right\} \right\} \delta y dx$$

$$\left\{ \int_A^B F dx \right\} \left\{ \frac{\partial G}{\partial y} - \frac{d}{ds} \frac{\partial G}{\partial y'} \right\} + \left\{ \int_A^B G dx \right\} \left\{ \frac{\partial F}{\partial y} - \frac{d}{ds} \frac{\partial F}{\partial y'} \right\} = 0$$

$$\int_A^B h_{ij} dx^j \int_A^B h_k^i dx^k = I$$

$$\therefore \int_A^B h_{ij} dx^j \left\{ \frac{\partial h_k^i dx^k}{\partial x^p ds} - \frac{d(h_k^i)}{ds} \right\} + \int_A^B h_k^i dx^k \left\{ \frac{\partial h_{ij} dx^j}{\partial x^p ds} - \frac{d(h_{ij})}{ds} \right\} = 0$$

$$\int_A^B$$

~~$$\int_A^B h_{ij} dx^j$$~~

$$\delta F\left(\int_A^B\right) = \frac{\partial F}{\partial \int_A^B} \left(\delta \int_A^B \right) = 0$$

$$\int_0^\pi \frac{d\varphi}{1 + \xi \cos \varphi} = \frac{\pi}{\sqrt{1 - \xi^2}}$$

$$\xi = \frac{\alpha \sqrt{z^2 - 1}}{1 - \alpha z}$$

~~$$(1 - \alpha z) \int_0^\pi \frac{d\varphi}{1 - \alpha \sqrt{z^2 - 1} \cos \varphi}$$~~

$$(1 - \alpha z) \int_0^\pi \frac{d\varphi}{1 - \alpha z \mp \alpha \sqrt{z^2 - 1} \cos \varphi} = (1 - \alpha z) \cdot \frac{\pi}{\sqrt{1 - 2\alpha z + \alpha^2 z^2 - \alpha^2 z^2 + \alpha^2}}$$

$$\int_0^\pi \frac{d\varphi}{1 - \alpha z \mp \alpha \sqrt{z^2 - 1} \cos \varphi} = \frac{\pi}{\sqrt{1 - 2\alpha z + \alpha^2}}$$

$$\int \mathcal{L} ds$$

$$I = \int_A^B \mathcal{L} ds$$

$$\delta(ds) = \frac{\Delta_{ij} dx^i d(\delta x^j)}{ds}$$

$$\delta(ds) = \hat{v} \cdot d(\delta \hat{e})$$

$$\delta I = \int_A^B \left[\mathcal{L} \frac{\partial \mathcal{L}}{\partial x^p} \delta x^p + \mathcal{L} \frac{\partial \mathcal{L}}{\partial v^q} \delta v^q + \mathcal{L} \left[\hat{v} \cdot d(\delta \hat{e}) \right] \right] ds$$

$$\delta v^q = \frac{dx^q}{ds} \hat{v} \cdot d(\delta \hat{e}) + \frac{d}{ds}(\delta x^q) = -v^q \left(\hat{v} \cdot \frac{d}{ds}(\delta \hat{e}) \right) + \frac{d}{ds}(\delta x^q)$$

$$\delta I = \int_A^B \left[e^{\mathcal{L}} \frac{\partial \mathcal{L}}{\partial x^i} \delta x^i + e^{\mathcal{L}} \frac{\partial \mathcal{L}}{\partial v^i} \left(\dot{v}^i + \frac{d}{ds} (\delta \dot{x}^i) \right) + e^{\mathcal{L}} \frac{\partial \mathcal{L}}{\partial m} \frac{d}{ds} (\delta x^i) + e^{\mathcal{L}} \left\{ \dot{v}^i \cdot \frac{d}{ds} (\delta \dot{x}^i) \right\} \right] ds \quad (25)$$